**Relaxation Time Approximation**

Basically going to copy and paste file from Stat Mech folder here. First, here’s a quick and dirty way to get the Diffusivity, taken from EM folder.

**Diffusivity from basic arguments**

Now let’s consider the electric diffusivity, D. This coefficient governs how a charge density gradient drives an electric current. To see how this comes about, consider a strip of metal with charge density profile ρ(x). I’m making it linear for illustrations sake, but it can be any desired curve.

A diagram of a line graph

Description automatically generated

At this position x, some particles will be flowing to the right, and some to the left. Those flowing to the right would’ve come from somewhere to the left of x. Specifically, they would’ve come from the point where they last collided with an impurity, a point at x-coordinate x-vxτ. The charge density of such *right* moving particles would be (1/2)ρ(x-v­xτ). The (1/2) is because at the coordinate x-vxτ (or any coordinate), we’d presume half the charges are moving to the left, and half the charges are moving to the right. And they’d be traveling to the right with velocity (in x-direction) vx. This would represent a current crossing the point x, of jright = (1/2)ρ(x-v­xτ)vx. Similarly, there would be a current passing our point x from right to left, given by jleft = (1/2)ρ(x+vxτ)vx. So the net current passing point x would be:



Note we see now that there is a net current going from right to left, because the density on the right is higher than that on the left, so more particles are crossing x from the left, than are crossing x from the right. Expanding in a Taylor series for small τ,



So we can see our Diffusion equation taking shape. We’ll perform a couple more manipulations to put this in 3D. So for a homogeneous system, we expect vx = vy = vz, and so vx2 = v2/3. So we can write,



and if we have density gradients in the other directions, we should expect,



Putting it all together, we have:



**Diffusivity from RTA equation (MFT approach)**

So going back to the Stat Mech folder and looking up the Classical NESM RTA (MF) file, we found that we could use the RTA equation and construct two self-consistent equations for the particle density and current, under a mean field approximation. The equations we found were,



where,



where d is the dimension. At T = 0 (or T << TF), we can approximate this for electrons as:



Note we can also write this as:



where ℓsc is the mean free path. Or if we’re dealing with classical particles – conduction electrons in semiconductors are an example – then we’d have:



Anyway, if we take our Boltzman equation and eliminate all fields and set time-dependence to zero, we clearly see that D, the proportionality between the current and the density gradient (sans e), is the diffusion constant. Now let’s go back to our equation and not set time-dependence to zero.



We can define a diffusion coefficient D(q,ω) as the proportionality constant (sans -e∇) between the current response, j(r,t) = Re[j(q,ω)eiq·r-iωt], n(r,t) = Re[n(q,ω)eiq·r-iωt], and j(q,ω) = -eD(q,ω)n(q,ω). But this time, we’ll need to use the time-dependent RTA equation (still no fields),



So plugging everything in,



So we have:



**Motion of diffusing particle**

Now let’s consider the time-independent case, and presume no external fields. So this is pure diffusion. Then we have:



Now divide both sides by e to get the number current density,



From the continuity, equation, we have:



We can combine these equations to get an equation for the density alone. So take the divergence of the first equation, presuming **D** is an isotropic tensor (**D** = D**1**)



and plug into the continuity equation, to get the diffusion equation.



Can solve this equation. Let’s presume spherical symmetry,



And we’ll take our initial condition to be the typical illustrative one: N particles all packed into the point r = 0, i.e., n0(r) = Nδ(r). Taking Laplace transform on time, we have:



For r ≠ 0, we have:



Let = r. Then,



So,



We don’t expect the density to ever exponentially grow with radius (at fixed time). So we can cross out the B term.



Then we have to determine A. We can determine this by employing the delta function thing. So let’s put our solution back into the full equation,



where in the last line we use Gauss’s law. Filling in our result,



Taking the ε → 0 limit,



So then our solution for is:



Now have to take inverse Laplace transform to get n(r,t). So,



Will deform the contour to wrap around the negative real s axis,



Okay so now we have a real integral to do,



Well let’s change variables s → s2.



and note that the following integral is the same, changing variables s → -s,



So we can add the two versions of the same integal together, and divide by two, to get:



Now we’ll integrate by parts,



And so we have:



This describes a Gaussian which is infinitely peaked at t = 0 (that’s the delta function initial condition), and spreads out over time. The standard deviation of Gaussian is:



and this gives us the rough radius encompassed by our particles as a function of time. Let’s sketch out the solution a different way. We can write our problem as:



Now take the Laplace transform on time. We have:



Now take the Fourier transform in space,



And now we’d do the inverse Laplace-Fourier transform.



Continuing,



And still going,



So once again we come to:

